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Exact expressions for a class of observables in supersymmetric gauge  
theories and in 2d CFT

SYNOPSIS

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Ատենախոսության թեման հաստատվել է Ա. Բ. Ալիխանյանի անվան Ազգային գիտական լաբորատորիայի գիտական խորհրդում:

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## Abstract

Instanton [1] partition function of  $N = 2$  supersymmetric gauge theory in  $\Omega$ -background admits exact investigation by localization methods [2-6]. In the case of generic  $\Omega$ -background instanton partition function is directly related to the conformal blocks of a 2d CFT (AGT correspondence) [7-11]. In this context the NS limit corresponds to the semi-classical limit of the related CFT [12-18]. We have extended some of the results of [18] to the case of generic  $\Omega$ -background corresponding to the genuine quantum conformal blocks. For technical reasons we restricted ourselves to the case of  $U(2)$  gauge groups. Restricting to the case of Liouville theory, starting from the second order differential equation satisfied by the multi-points conformal blocks including a degenerate field  $V_{-b}$  [19] we derived the analogues equation satisfied by the gauge theory partition function with  $Q$  operator insertion. Then we showed that this equation leads to a mixed linear difference differential equation for  $Q$  operators which is a direct generalization of the  $T = Q$  equation from NS limit to the case of generic  $\Omega$ -Background.

The 4d  $N = 2$  gauge theories have natural uplift to 5 dimensions. Embedding  $N = 2$  gauge theory in  $\Omega$ -background was instrumental in all developments related to the instanton counting with the help of equivariant localization technics. In fact, the geometric meaning of  $\Omega$ -background is more transparent in 5d theory compactified on a circle. One simply considers a 5d geometry fibered over a circle of circumference  $L$  so that the complex coordinates  $(z_1, z_2)$  of the (four real dimensional) fiber get rotated along the circle as:  $z_1 \rightarrow \exp(iL\epsilon_1)$ ,  $z_2 \rightarrow \exp(iL\epsilon_2)$  accompanied with suitable  $R$ -symmetry and gauge rotations [3,6].  $\epsilon_{1,2}$  are the  $\Omega$ -background parameters. In paper [11] it is shown that not only the partition function, but also a more refined quantity, namely the expectation value of the  $Q$ -observable, can be computed in closed form. It was shown in [20] that the analog of Baxter's  $Q$  operator in purely gauge theory context naturally emerges in Nekrasov-Shatashvili limit ( $\epsilon_2 = 0$ ) [21] as an entire function whose zeros are given in terms of an array of "critical" Young diagrams, namely those, that determine the most important instanton configuration contributing to the partition function. In [22] an algebraic construction of a RG domain wall for the unitary minimal CFT models was proposed and was

shown that the results agree with those of the leading order perturbative analysis performed by A. Zamolodchikov in [23]. In the paper [22] Gaiotto suggests that a similar construction should be valid also for more general coset CFT models. The  $N = 1$  minimal superconformal CFT models [24–26], which are the main subject of paper [III], are among these cosets. We specialize Gaiotto's proposal to the case of the minimal  $N=1$  SCFT models. The method we use is based directly on the current algebra construction and, in this sense, is more general than the one originally employed by Gaiotto for the case of minimal models. Namely he heavily exploited the fact that the product of successive minimal models can be alternatively represented as a product of  $N = 1$  superconformal and Ising models. We explicitly calculated the mixing coefficients for several classes of fields and compare the results with the perturbative analysis of [27,28] finding a complete agreement.

#### Timeliness and relevance

All papers[I-III] that form the basis of this thesis are located on a crossroad between the fields of Supersymmetric field theories, gauge field theories and instanton calculus. Supersymmetry is a space-time symmetry discovered (rediscovered) in the 1970's independently by Gervais and B. Sakita (in 1971) [29], Golfand and Likhtman (also in 1971) [30], and Volkov and Akulov (1972) [31]. Although its existence in nature is not proved or disproved it plays a major role in theoretical physics. One of the reasons of its popularity is the Coleman-Mandula theorem [32] which elevates supersymmetry to the status of the single possible extension of the Poincare group, assuming some natural constraints. The existence of the conformal symmetry is a loophole in this theorem, where all particles are massless.

Gauge symmetry was first present in Maxwell's famous work on electrodynamics "A Dynamical Theory of the Electromagnetic Field" in 1864-65 [33], the more modern formulation was popularized by Pauli in 1941 [34]. Nowadays gauge fields are used to describe three of the four fundamental forces, and one hardly needs any other reasons to study them, furthermore they are present in broad areas of pure mathematics and theoretical physics such as differential geometry and Gravity.

Instantons are a more specific area of research and occur in various situations and contexts like in the calculation of tunneling effects of vacuum states in quantum field theories or in calculations of path integrals in the semiclassical limit [35].

#### Aim of the dissertation

- Derive a mixed difference-differential relation for Q-operator for SU(2) linear quiver theory.
- For the  $N=1$ , 5D linear quiver theory derive the analogue of Baxters T - Q equation and its solutions.
- Calculate the field mixing coefficients for  $N=1$  SCFT models  $SM_p$  and  $SM_{p-2}$ .
- Derive the T - Q equation and its solutions for 4D quiver out of the 5D linear quiver theory.

#### Novelty of the work

In this work the following new results were obtained:

1. We show that an appropriate choice of parameters in  $A_{r+1}$  linear quiver theory with  $U(n)$  gauge groups is equivalent to insertion of the analog of Baxters Q-operator into the partition function of a theory with one gauge node less  $A_r$  theory with generic parameters.
2. Restricting to the case of Liouville theory, starting from the second order differential equation satisfied by the multi-points conformal blocks including a degenerate field  $V_{\frac{b}{2}}$  we've derived the analogues equation satisfied by the gauge theory partition function with Q operator insertion. We also showed that this equation leads to a mixed linear difference-differential equation for Q operators which is a direct generalization of the

T-Q equation from NS limit to the case of generic  $\Omega$ -Background.

3. By exploiting the fact that the product of successive minimal models can be alternatively represented as a product of  $N = 1$  superconformal and Ising models. We explicitly calculate the mixing coefficients for  $N=1$  SCFT models  $SM_p$  and  $SM_{p-2}$ .
4. The linear quiver with  $U(1)$  in 5d setting is analyzed. The corresponding T-Q difference equations as well as their solutions in closed form are found. The solution is expressed in terms of generalized Appel's function. As a limit the same is found for 4d.

#### Practical value

Supersymmetric gauge theories and superconformal field theories play important role for the various problems in condensed matter, String theory, AdS/CFT and AGT correspondences. Our findings can find an application or extension in all these topics.

#### Main points to defend

Main points to defend are:

1. We show that an appropriate choice of parameters in  $A_{r+1}$  linear quiver theory with  $U(n)$  gauge groups is equivalent to the insertion of the analog of Baxters Q-operator into the partition function of a theory with one gauge node less  $A_r$  theory with generic parameters. It is also true that only a subset of special diagrams contribute to the partition function in  $A_{r+1}$ , these  $n$ -tuple of diagrams consist of a row diagram and  $n - 1$  empty diagrams. We found that to the Nekrasov partition

functions corresponding to  $N = 1$  SLFT conformal blocks in the light asymptotic limit contribute only Young diagrams having one row and one column.

2. Restricting to the case of Liouville theory, starting from the second order differential equation satisfied by the multi-points conformal blocks including a degenerate field  $V_{\frac{b}{2}}$  we have derived the analogues equation satisfied by the gauge theory partition function with Q operator insertion. We also showed that this equation leads to a mixed linear difference differential equation for Q operators which is a direct generalization of the T-Q equation from NS limit to the case of generic  $\Omega$ -Background.
3. By exploiting the fact that the product of successive minimal models can be alternatively represented as a product of  $N = 1$  superconformal and Ising models. We explicitly calculate the mixing coefficients for  $N=1$  SCFT models  $SM_p$  and  $SM_{p-2}$ . We calculate the mixing coefficients for the several classes of local fields in the case of the supersymmetric RG flow using RG domain wall proposal. Then we compare this with the perturbation theory results available in the literature finding a complete agreement.
4. The linear quiver with  $U(1)$  in 5d setting is analyzed. The corresponding T-Q difference equations as well as their solutions in closed form are found. The solution is expressed in terms of generalized Appel's function, which can be represented as Heine's basic hypergeometric series. The 4d linear quiver theory is a natural limit of the 5d theory. Analog T-Q difference equations are deduced with their corresponding solutions, which are Appel's  $F_1$  functions generalization for many variables. The normalization factors for both 4d and 5d cases are found.

## Length and structure of the dissertation

The thesis is divided into five parts. The first part is dedicated to some prerequisites on which we draw on later. The following three chapters describe our works, which are followed by the bibliography.

## Content of the dissertation

The thesis starts with a short introduction. The first chapter explores a variety of topics such as conformal symmetry, ADHM construction and supersymmetry. In the next chapters we describe the original works.

## Chapter 1

Chapter 1 starts with a short introduction of conformal symmetry(1.1). Here the importance of the symmetry are highlighted, continued by the geometric and formal definitions. Followed by the derivation of the algebra generators with space-time dimensions bigger than two. The generators are explained and paired with the corresponding group members. Afterwards the Witt and Virasoro algebras are introduced. Then a quick outline of the differences between different space time dimensions is given, followed by the necessary references for this part.

Because our main interest is in the application of instantons in gauge theories at the beginning of section 1.2 a short overview of path integrals and their connection with Feynman diagrams and non-perturbative effects is given. Further the definition of instanton is mentioned, with the deliberate choice of Euclidean metrics. The process of changing to Euclidean metric is also explained. In section 1.2.1 the system of a double well potential is discussed. The main goal here is to illustrate a situation where instantons arise in a well-known system. Then, it is argued that the shift in vacuum states is described by instantons, also by explicit

calculations this effect is clarified. In the end the tunneling amplitude is written which operates as expected.

In section 1.2.2 an introduction of instantons in Yang-Mills theories is given, by presenting the action, equations of motion and the Bianchi identities, which makes possible to properly define instantons and anti-instantons. Also a discuss of the benefits of Wick rotation and its practicality and the differences between Euclidean space-time over Minkowski space-time in this setting is given, which in itself is a reacquiring theme in supersymmetry. Then the instanton number and the Chern character are defined, it is also indicated that instanton solutions are an essential part in approximations of path integrals and non-perturbative effects.

The section 1.2.3 is devoted to the Clifford algebra and its representations. The definition of Clifford algebra in Euclidean and Minkowski spaces are given continued by representations of the algebra for two dimensions, four dimensions (which are the Dirac gamma matrices) and in six dimensions (these are the famous 't Hooft symbols [11]), the representations are given in both Minkowski and Euclidean spaces. At the end a scheme for construction of arbitrary dimensional representations in Euclidean space is illustrated.

In 1.2.4 the connection between Young diagrams to partitions is given. Euler's famous equation is also mentioned, with a hint on how to prove it. This section is a tribute to actual calculations done in [I,III].

Next, in section 1.2.5, the ADHM construction [12] is introduced, which is a method for constructing a self-dual field strength. Also, the moduli space for instantons with instanton number  $k$  is defined, and its dimension is indicated. By a straight check the correctness of ADHM is confirmed. At the end of this section the BPSI [13] instanton is introduced by showing that it is a special case of the ADHM construction.

In section 1.3 the Lorentz algebra and its representations are discussed. The algebra of Lorentz transformations is given, the more familiar space rotations and boosts are also defined. The definitions of representations and equivalent representations are given. Also the notion of irreducible representations is highlighted. Then the direct sum and direct product, as methods to construct higher dimensional representations, are reviewed. As an example the  $4 \otimes 4$  representation and its reduction to a direct sum of irreducible representations is illustrated. In a simplistic

fashion the notions of Hodge dual, tensor representations and spinor representation are discussed. At the end, the construction of irreducible representation via the  $SU(2) \otimes SU(2)$  covering group is shown.

1.3.1 is devoted to Majorana spinors. This review is meaningfully divided into two, first the simple connection between the Dirac equation and Majorana spinors is described. The second part is devoted to the formally correct illustration of Majorana spinors. A basis for  $4 \times 4$  matrices is constructed out of the gamma matrices. The  $\gamma_5$  matrix and with it the Weyl spinors are defined. Then by the construction of some auxiliary operators the Majorana spinors are defined. A proof of the contradiction of the Weyl and Majorana conditions is derived.

Supersymmetry has a central role in theoretical physics. One way to see its importance and give a introduction to it is to look at the Coleman-Mandula theorem. In section 1.4.1 the Coleman-Mandula theorem is given and its implications are explored by a simple thought experiment. Then in a toy theory of two scalar fields it is argued that additional generators of internal symmetries must be Lorentz scalars. Then by adding a fermion field with interaction it is argued that the only extension of Poincar algebra is a spin one half conserved current, which are the generators of supersymmetry.

Next in 1.4.2 the superspace is introduced as the natural upgrade of Minkowski space with Grassmann coordinates. Necessary differential and integral relations are given.

In the following section (1.4.3) the superfield is introduced as a field on superspace. By expanding the superfield in Grassmann coordinates a connection is established between superfields and usual field on Minkowski space. The distinction of fermionic and bosonic superfields is established. The notions superderivatives and supercharges is also reviewed with their corresponding anticommutative and commutation rules. Then the chiral and vector superfields are introduced, the gauge superfield is illustrated as a natural sub case of the vector superfield. By gauge fixing the Wess-Zumino field is detached.

## Chapter 2

Chapter 2 is based on paper [I] and paper [37]. Linear quiver  $N = 1$  5D gauge theory in  $\Omega$  background is considered. It is shown that under certain restrictions on the VEV's of the adjoint scalar field corresponding to the first node, only the array of Young diagrams, such that the first diagram has a single column only the others are empty, contribute to the partition function. Furthermore, it is proved that this partition function in a simple way is related to the expectation values of Baxter's Q operator (at specific discrete values of the spectral parameter) in the gauge theory with the special node removed. Using known expression of the partition function in the  $U(1)$  quiver, Baxter's T-Q difference equations are established and explicit expressions for the VEV of the Q operator in terms of generalized q-deformed Appel's functions is found. Finally, the corresponding expressions for the 4D limit are derived.

The chapter is organized as follows.

Section 2.1 is introductory.

In section 2.2 is a short review of 5d linear quiver gauge theory; the Nekrasov partition function and important observables  $Q$  and  $y$  are introduced.

In section 2.3 an extended quiver with specific parameters at the extra node is introduced and its relation to the  $Q$ -observable is analyzed.

Section 2.4 specializes to the case of  $U(1)^r$  theory. Difference equations  $Q$ -observable are derived. Explicit expressions for the  $Q$  observable in terms of generalized Appel and hypergeometric functions are found.

In section 2.5 through dimensional reduction, corresponding difference equations and their solutions for the 4d theory are found. In the end (2.7,2.8) some technical details, used in the main text, are presented.

## Chapter 3

Chapter 3 is based on paper [II]. In this short chapter using AGT correspondence we express simplest fully degenerate primary fields of Toda field theory in terms an analogue of Baxter's Q-operator naturally emerging in  $N = 2$  gauge theory side. This quantity can be considered as a

generating function of simple trace chiral operators constructed from the scalars of the  $N - 2$  vector multiplets. In the special case of Liouville theory, exploring the second order differential equation satisfied by conformal blocks including a degenerate at the second level primary field (BPZ equation) we derive a mixed difference-differential relation for  $Q$ -operator. Thus we generalize the  $T$ - $Q$  difference equation known in Nekrasov-Shatashvili limit of the  $\Omega$ -background to the generic case.

Section 3.1 is introductory.

In section 3.2 we show that an appropriate choice of parameters [29] in  $A_{r+1}$  linear quiver theory with  $U(n)$  gauge groups is equivalent to insertion of the analog of Baxters  $Q$ -operator into the partition function of a theory with one gauge node less  $A_r$  theory with generic parameters. In the 2d CFT side such special choice corresponds to insertion of a degenerated primary field in the conformal block [29].

In section 3.3 restricting to the case of Liouville theory, starting from the second order differential equation satisfied by the multi-points conformal blocks including a degenerate field  $V_{\frac{b}{2}}$  [19] we derive the analogues equation satisfied by the gauge theory partition function with  $Q$  operator insertion. Then we show that this equation leads to a mixed linear difference-differential equation for  $Q$  operators which is a direct generalization of the  $T$ - $Q$  equation from NS limit to the case of generic  $\Omega$  Background. Finally, we summarize our results and discuss a couple of further directions which we think are worth pursuing.

## Chapter 4

Chapter 4 is based on paper [III]. We specify Gaiotto's proposal for the RG domain wall between some coset CFT models to the case of two minimal  $N-1$  SCFT models  $SM_p$  and  $SM_{p-2}$  related by the RG flow initiated by the top component of the Neveu-Schwarz superfield  $\Phi_{1,3}$ . We explicitly calculate the mixing coefficients for several classes of fields and compare the results with the already known in literature results obtained through perturbative analysis. Our results exactly match with both leading and next to leading order perturbative calculations. The chapter is organized as follows:

Section 4.1 is introductory.

Section 4.2 is a brief review of the 2d  $N - 1$  superconformal field theories.

Section 4.3 is devoted to the description of the coset construction of  $N - 1$  SCFT. Of course everything here is well known, our purpose here is to fix notations and list the relevant formulae in a form, most convenient for the further calculations.

In section 4.4 we formulate Gaiotto's general proposal for a class of coset CFT models.

Section 4.5 is the main part of our paper. We explicitly calculate the mixing coefficients for the several classes of local fields in the case of the supersymmetric RG flow discussed above using RG domain wall proposal. Then we compare this with the perturbation theory results available in the literature finding a complete agreement. [7]

## Conclusions

In Liouville theory, starting from the second order differential equation satisfied by the multi-points conformal blocks including a degenerate field  $V_{\frac{b}{2}}$  we've derived the analogues equation satisfied by the gauge theory partition function with  $Q$  operator insertion. We explicitly calculate the mixing coefficients for  $N-1$  SCFT models  $SM_p$  and  $SM_{p-2}$ . We found topological defects for  $N = 1$  super Liouville field theory. The linear quiver with  $U(1)$  in 5d setting is analyzed. The corresponding  $T$ - $Q$  difference equations as well as their solutions in closed form are found. The solution is expressed in terms of generalized Appel's function. As a limit the same is found for 4d.

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  37. G. Poghosyan, VEV of Q-operator in U(1) linear quiver 5d gauge theories. arXiv:1801.04303.

### Publications list

- I. G. Poghosyan, VEV of Q-operator in U(1) linear quiver 4d gauge theories. Armenian Journal of Physics, 11 (1). pp. 34–38. ISSN 1829-1171
- II. G. Poghosyan, R. Poghossian, "VEV of Baxter's Q-operator in  $N=2$  gauge theory and the BPZ differential equation" JHEP 1611 (2016) 058 .
- III. G. Poghosyan, H. Poghossian, "RG domain wall for the  $N=1$  minimal superconformal models" JHEP 05(2015)043 .

**Ճշգրիտ արտահայտություններ սուպերսխեմորիկ տրամաչափային տեսություններում և երկչափ ԿՂՏ-ում մի դաս մեծությունների համար**

**Անիտիում**

Չ-ֆոնում  $N = 2$  սուպերսխեմորիկ տրամաչափային տեսության ինստանտոնային [1] վիճակրագական գումարը ենթակա է տեղայնացման մեթոդներով [2-6] ճշգրիտ հետազոտման: Ընդհանուր Չ-ֆոնի դեպքում, ինստանտոնային վիճակրագական գումարը անմիջականորեն կապված է երկչափ ԿՂՏ կոնֆորմ բյուկներին հետ (AGT համապատասխանություն) [7-11]: Այս շրջանակում NS սահմանը համապատասխանում է հարակից ԿՂՏ-ի [12-18] կիսադասական սահմանին: Մենք ընդլայնել ենք [18]-ի որոշ արդյունքները ընդհանուր Չ-ի ֆոնի դեպքում, որը համապատասխանում է բուն քվանտային կոնֆորմ բյուկներին: Տեխնիկական պատճառներով մենք սահմանափակել ենք հաշվարկները  $U(2)$  տրամաչափային խմբի համար: Մկսելով երկրորդ կարգի դիֆերենցիալ հավասարումից, որին բավարարում է բազմակի կետային կոնֆորմ բյուկները որոնցում ներառվել է այլասերված  $V_{\frac{2}{2}}$  [19] դաշտը, սահմանափակվելով Լիովիլյան տեսությամբ, մենք դուրս ենք բերել նմանատիպ հավասարումը, որը բավարարում է տրամաչափային տեսության վիճակրագական գումարը  $Q$  օպերատորի տեղադրումից հետո: Այնուհետև, մենք ցույց ենք տվել, որ այս հավասարումը հանգեցնում է  $Q$  օպերատորների խառը զծային տարբերությունների-դիֆերենցիալ հավասարման, որը  $T-Q$  հավասարման ուղղակի ընդհանրացումն է NS սահմանից մինչև ընդհանուր Չ-ֆոնի:  $4d$   $N = 2$  տրամաչափային տեսությունները ունեն 5 չափանի բնական ընդհարացում: Էկվիվարիանտ տեղայնացման տեխնիկայի օգնությամբ ինստանտոնային հաշվարկի զարգացման համար Չ-ֆոնում ներդրված  $N = 2$  տրամաչափային տեսությունը ունեցել է հիմնարար օժանդակող դեր: Փաստորեն, Չ-ֆոնի իմաստը ավելի հստակ է շրջանագծի վրա կոմպակտացված  $5d$  տեսության մեջ: Կարելի է դիտարկել  $4D$  շերտավորված երկրաչափություն, որի

հիմքը իրենից ներկայացնում է  $L$  շառավղով շրջանագիծ, այնպես որ  $(z_1, z_2)$  քառաչափ իրական շերտի տարածությունը պատկերվում է շրջանի շուրջը՝  $z_1 \rightarrow \exp(iLz_1)$ ,  $z_2 \rightarrow \exp(iLz_2)$ , այն ուղեկցվում է համապատասխան  $R$ -սխեմորիաի և տրամաչափային ձևափոխության հետ[3,6]:

Եւ-ը Չ-ֆոնի պարամետրերն են: Մենք ցույց ենք տվել, որ ոչ միայն վիճակրագական գումարը, այլ նաև  $\Omega$ -ի սպասման արժեքը կարելի է ճշգրիտ հաշվել, որը ավելի բարդ մեծություն է: Ցույց է տրվել, որ զուգահեռ տեսության համատեքստում՝ Նեկրասով-Շատաշվիլի սահմանում ( $\epsilon_2 = 0$ )[21] Բակստերի  $Q$  օպերատորին նմանարիպ օպերատոր է առաջանում, որպես ամբողջ ֆունկցիա, որի գրոները տրվում են «կրիտիկական» Յունգի դիագրամների մասիվի տեսքով, որոնք որոշում են վիճակրագական գումարին նպաստող կարևորագույն ինստանտոնային կոնֆիգուրացիան: [22]-ում ունիտար մինիմալ ԿՂՏ մոդելների համար  $RG$  տիրույթային պատի հանրահաշվային կառուցումն առաջարկվել և ցույց է տրվել, որի արդյունքները համընկնում են Ա. Ջամոլոդչիկովի կողմից կատարված խտտորումային վերլուծությանը[23]: [22] հողվածում Գայտոն ենթադրում է, որ նման կառույցը պետք է լինի նաև վավեր է ավելի ընդհանուր կոնստային  $CFT$  մոդելների համար:  $N = 1$ , մինիմալ, սուպերկոնֆորմալ, ԿՂՏ[24-26] մոդելները, որոնք [III] հողվածի հիմնական թեմաներն են, այդպիսի կոնստանտներից են: Մենք իրականացնում ենք Գայտոնի առաջարկը  $N = 1$  SCFT մոդելի դեպքում: Օգտագործվող մեթոդը ուղղակիորեն հիմնված է հոսանքի հանրահաշվի վրա, և այս իմաստով ավելի ընդհանուր է, քան մինիմալ մոդելների դեպքում Գայտոնի կողմից ի սկզբանե գործածվող մեթոդը: Մասնավորապես, նա հաճախակի շահագործել է այն փաստը, որ հաջորդական մինիմալ մոդելների արտադրյալը կարելի է փոխարինել, որպես  $N = 1$  սուպերկոնֆորմալ և Իզինգի մոդելների արտադրյալով: Մենք ակնառու հաշվարկել ենք, դաշտերի մի քանի դասերի համար, խառնման գործակիցները և արդյունքները համեմատել ենք խտտորումանին վերլուծությամբ ստացված [27,28] ի արդյունքների հետ, գտնելով ամբողջական համաձայնություն:

Погосян Габриел Рубикович  
Точные выражения для наблюдаемых в суперсимметричных  
калибровочных теориях и в двумерных КТП

Резюме

Инстантонная статсумма [1]  $N = 2$  суперсимметричной калибровочной теории в  $\Omega$ -фоне допускает точное исследование методами локализации [2-6]. В случае обобщенной  $\Omega$ -фона инстантонная статсумма непосредственно связана с конформными блоками 2d КТП (соответствие AGT) [7-11]. В этом контексте предел NS соответствует полуклассическому пределу связанного КТП [12-18]. Мы расширили некоторые результаты из [18] на случай общего  $\Omega$ -фона, соответствующих подлинным квантовым конформным блокам. По техническим причинам мы ограничимся случаем калибровочных групп  $U(2)$ . Ограничиваясь случаем теории Лиувилля, начиная с дифференциального уравнения второго порядка, удовлетворяемого многоточечными конформными блоками, включая вырожденное поле  $V_{-\frac{b}{2}}$  [19], выводится аналоговое уравнение, удовлетворяющее статсуме калибровочной теории с  $\Omega$  вставкой. Затем мы показали, что это уравнение приводит к смешанному линейному разностно-дифференциальному уравнению для операторов  $\Omega$ , являющемуся прямым обобщением уравнения T-Q от предела NS к случаю общей Q-фона. 4d  $N = 2$  калибровочные теории имеют естественное поднятие до 5 измерений. Вложение  $N = 2$  калибровочной теории на  $\Omega$ -фоне сыграло важную роль во всех событиях, связанных с подсчетом инстантонов с помощью эквивариантной техники локализации. На самом деле геометрический смысл  $\Omega$ -фона более прозрачен в теории 5d, компактифицированной на окружности. Просто рассмотрим 5d-геометрию, расслоенную над окружностью окружности  $L$ , так что комплексные координаты  $(z_1, z_2)$  (четырёхмерного) слоя вращаются по кругу как:  $z_1 \rightarrow \exp(iL\epsilon_1)$ ,  $z_2 \rightarrow \exp(iL\epsilon_2)$  с соответствующими R-симметриями и калибровочными вращениями [3,6].  $\epsilon_1, \epsilon_2$  является параметры  $\Omega$ -фона. Мы показываем, что не только

статистическая сумма, но и более тонкая величина, а именно ожидаемое значение  $\Omega$  наблюдаемый, можно вычислить в замкнутом виде. В [20] показано, что аналог оператора Бакстера  $Q$  в контексте калибровочной теории естественно возникает в пределе Некрасова-Шаташвили ( $\epsilon_2 \rightarrow 0$ ) [21] как целая функция, нули которой задаются в терминах матрицы " критических диаграмм Юнга, а именно те, которые определяют самую важную конфигурацию инстантона, способствующую статсуме. В [22] алгебраической конструкции доменной стенки PG для унитарных минимальных моделей КТП было предложено и показано, что результаты согласуются с результатами пертурбативного анализа первого порядка, выполненного А. Замолодчиковым в [23]. В статье [22] Гайотто предполагает, что аналогичная конструкция должна быть действительной и для более общих смежных моделей КТП.  $N = 1$  минимальные суперконформные КТП модели [24-26], которые являются одним из основных предметов работы [11], относятся к числу этих смежных классов. Мы специализируемся на предложении Гайотто к случаю минимальных моделей  $N = 1$  СКТП. Используемый нами метод основан непосредственно на построении ток алгебры и в этом смысле более общий, чем тот, который первоначально использовался Гайотто для случая минимальных моделей. Он в значительной степени использовал тот факт, что произведение последовательных минимальных моделей может быть альтернативно представлено в виде произведения  $N = 1$  суперконформных и Изигских моделей. Мы явно вычисляем коэффициенты смешивания для нескольких классов полей и сравниваем результаты с пертурбативным анализом [27,28] нахождения полного согласия.

Gaiotto